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A Robust and Adaptive Force/Position Control for Two Cooperating Robot Arms Under Uncertainty

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This paper presents a motion coordination of a two-cooperating robot arm when there are unknown system parameters and bounded input disturbances. The order of the model of the two-arm system is reduced. To control this, a force/position control scheme based on an inverse dynamics control scheme is devised. On the top of the control scheme, an adaptive control scheme to take care of parametric uncertainties, and a robust control scheme to compensate coupling forces between two arms and input disturbances are devised. The adaptive and the robust control scheme are derived based on a devised Lyapunov function. The adaptive control algorithm is practical since it does not require the feedback of the second derivative of joint angles and interacting forces. The robust control scheme guarantees that the tracking error of the leader arm and the interacting forces between two arms are confined in a certain region. Numerical examples using dual 3 degree of freedom robot arm are shown.

Key Words : Robust Control, Adaptive Control, Force/Position Control, Parametric Uncertainties, Lyapunov's Second Method

1. Introduction

There are many applications of robots in assembly automation and flexible manufacturing systems such as material handling, maintenance, etc. These applications require coordinated operation of robot arms which are kinematically and dynamically coupled. For this reason, many coordinated control schemes are devised and some of them are based on a master/slave (or leader/follower) control which are devised by Ishida (1977), Tarn (1986), Arimoto (1987), and Ro (1991). Ishida (1977) and Ro (1991) devised a PID and a computed torque scheme to control the positions and velocities of the leader arm, respectively, while the slave arm is controlled to follow the leader arm by the force feedback. Arimoto (1987) devised a control law based on Lyapunov Direct Method to ensure zero steady state operation of the coordinator. Tarn (1986) devised a dynamic coordinator which generates the control action based on relative force/torque errors between the two arms where a linearlization scheme is also applied. The kinematic relations between two coordinated robots in handing several different shape of object is studied by Luh (1987). In the presense of parameter uncertainties, actuator nonlinearlities, and bounded disturbance, a robustness analysis of a two-interacting system is shown by Ro (1989). Hayati (1986) and Uchiyama (1988) devised hybrid position/force control method where the leader arm is coordinated to unconstrained direction while the force control is applied to constrained directions.

In the presence of unknown or changing system parameters, the system dynamics may show undesirable overshoot or instability. To solve this problem, several adaptive control schemes are formulated in robotics. Craig (1986) and Middleton (1988) devised adaptive control schemes based on a computed torque scheme. The scheme requires the feedback of the acceleration and the inversion of the inertia matrix of robot arm which

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is function of the estimated parameters. By using the skew symmetric relationship between the derivative of the inertia matrix with respect to time and the Corliolis and centripetal term, the structure of the manipulator dynamics is simplified via feedback by Saddegh (1987) and Slotin (1986). As a result, a computationally simple adaptive control algorithm is formulated. In addition to the problem of the parametric uncertainties, Reed (1988) studied the robustness of the adaptive control scheme with respect to system uncertainties such as unmodeled dynamics, parameter variations, etc. However, all the adaptive control schemes mentioned above are devised for non-cooperating robot arms.

Seraji (1988) devised an adaptive position/force control approach to the dual-arm problem. By employing an adaptive PID structure, knowledge of the mathematical model of the system is not required. The coupling effects between the manipulators, through the common payload, are modeled as disturbances in the position and force equations, which are then compensated for in the adaptation rule. By Choi (1992) and Hu (1989), in the presence of parametric uncertainties of multiple interacting robot arms, adaptive algorithms are devised, and stability analyses are shown.

When there exist disturbances such as uncertain inputs, noises, etc., the response of the system shows undesirable system behavior and tracking errors. In order to solve these problems, a robust control scheme to take care of disturbances is devised by Chen (1986) and Leitmann (1979). Cai (1990) and Chen (1991) applied the control schemes to robot manipulators. Especially, by Chen (1991), in conjunction with a computed torque method, nonadaptive robust versus robust adaptive control is applied to manipulators, and both schemes are compared to each other by the simulation results.

In this paper, a robust and adaptive control scheme in conjunction with an inverse dynamics control scheme for motion coordination of a two-cooperating robot arm is presented. Based on an inverse dynamics control scheme, the leader arm is coordinated by a position control scheme and the interacting forces between two arms are regulated by a force regulator. However the parameters of the system are unknown such that an adaptive control algorithm is devised, which is improved from Ortega's (1988) algorithm. The devised algorithm is practical because the feedback of the acceleration of the joint angles and the second derivative of force is not required. To take care of the coupling forces between two arms and input disturbances, a robust control scheme adapted from Chen (1986) and Leitmann (1979) is applied. Numerical examples for the devised control scheme is shown.

In Section 2, the leader arm dynamics incorporate the object dynamics such that the order of the two-arm system dynamics is reduced. In Section 3, based on the reduced order model, a position control scheme by the inverse dynamics control scheme is applied to coordinate the leader arm. In section 4, the dynamics of the follower arm are modified as force dynamics such that a direct force control scheme is applied based on an inverse dynamics control scheme. In section 5, to take care of the parametric uncertainties as shown in Section 3 and 4, a practical adaptive control scheme based on Lyapunov's Second Method is devised. In addition to this, a nonlinear robust controller is designed to counteract the coupling forces between two arms and bounded input disturbances. In Section 6, a numerical example using dual 3 degree of freedom robot arm is shown to validate the devised control scheme.

2. Two-Arm Dynamics

In this paper, the dynamics of the object is incorporated into that of leader arm and considered as a portion of the arm dynamics. Therefore, $(3n\times 1)$ size of two-arm dynamic systems are reduced to $(2n\times 1)$ size of dynamic systems. This is shown by Choi (1992). For convenience, the leader arm is called arm b and the follower arm is called arm a in this paper. The equations of motion for two robot arms coordinating an object can be expressed as the following :

$$H_a(q_a)\ddot{q}_a + C_a(q_a, \dot{q}_a) = \tau_a + J_a^T(q_a)F_a \quad (1)$$

$$H_b(q_b)\ddot{q}_b + C_b(q_b, \dot{q}_b) = \tau_b + J_b^T(q_b)F_b \quad (2)$$

where $q_a(q_b)$ is the $n\times 1$ joint angle vector for arm a(arm b); $\tau_a(\tau_b)$ is the $n\times 1$ joint torque vector for arm a(arm b); $H_a(H_b)$ is the $C_a(C_b)$ is the nonlinear force vector of size $n\times 1$ including gravity term; and $J_a(J_b)$ is the $n\times n$ Jacobian matrix of arm a(arm b). $F_a(F_b)$ is an $n\times 1$ vector representing the forces and the moments at the point of interaction between arm a(arm b) and the object. The equations of motion for the object can be expressed as the following:

$$M_o\ddot{x}_o+Q_o(\dot{x}_o, x_o)=-L_a^TF_a-L_b^TF_b \quad (3)$$

where x_o is an $n\times 1$ vector representing the position and orientation of the object center in the inertial space; M_o is an 6×6 inertia matrix of the object; Q_o is the 6×1 nonlinear force vector of the object including the gravity term. $n\times n$ matrix $L_a(L_b)$ represents the Jacobian matrix associated with a finite length between the center of the object and the interaction point $a(b)$. The object is assumed to be rigidly grasped by arm b. Arm b and the object are kinematically related such that the velocity and the acceleration of the mass center of the object are expressed in terms of the joint coordinates of robot arm b by the Jacobian matrix J_b as the following:

$$\dot{x}_o=L_b^{-1}J_b\dot{q}_b,$$
$$\ddot{x}_o=\dot{L}_b^{-1}J_b\dot{q}_b+L_b^{-1}\dot{J}_b\dot{q}_b+L_b^{-1}J_b\ddot{q}_b \quad (4)$$

By substituting Eq. (4) into Eq. (3), the object dynamics are expressed in joint coordinates as

$$M_oG(\dot{q}_b, \ddot{q}_b)+Q_{ob}(\dot{q}_b, q_b)$$
$$=-L_b^TF_b-L_a^TF_a \quad (5)$$

where

$$G(\dot{q}_b, \ddot{q}_b)=\dot{L}_b^{-T}J_b\dot{q}_b+L_b^{-T}\dot{J}_b\dot{q}_b+L_b^{-T}J_b\ddot{q}_b,$$

Q_{ob} represents Q_o in the joint coordinates of arm b. Expressing Eq. (5) with respect to interacting force F_b and substituting it into Eq. (2) yield

$$H_b^*\ddot{q}_b=\tau_b+C_{ob}-\gamma F_a \quad (6)$$

where

$$H_b^*=H_b+J_b^TL_b^{-T}M_oL_b^{-1}J_b,$$
$$\gamma=J_b^TL_b^{-T}L_a^T, \text{ and}$$
$$C_{ob}=-J_b^TL_b^TQ_{ob}-C_b$$
$$-J_b^TL_b^{-T}M_o(\dot{L}_b^{-1}J_b+L_b^{-T}\dot{J}_b)\dot{q}_b,$$

where H_b^* represents the inertia matrix of arm b dynamics incorporating that of the object. C_{ob} represents the nonlinear force vector of arm b dynamics incorporating that of the object. γ represents the Jacobian matrix reflecting the interacting force, F_a in joint space of arm a to arm b.

3. A Position Control of Leader Arm via an Inverse Dynamics Control for Two-Arm Coordination under Uncertainty

The open-loop arm b dynamic equations show that the dynamics of arm b are disturbed by the interacting force, F_a. An inverse dynamics control is proposed to achieve position control of arm b such that τ_b is composed as

$$\tau_b=\bar{H}_b^*(\ddot{q}_{db}-K_{db}\dot{E}-K_{pb}E-U_b)$$
$$+\bar{C}_{ob}+D_b(t) \quad (7)$$

where $E=q_b-q_{db}(t)$, $q_{db}(t)$ is a bounded desired input signal; $\bar{H}_b^*$ and $\bar{C}_{ob}$ are initially well estimated such that it is no longer a function of the estimate of the unknown parameters by the adaptive control algorithm. For this reason, in this paper $\bar{H}_b^*$ is called a known estimate of the inertia matrix H_b^*; $\bar{C}_{ob}$ is also called a known estimate of the nonlinear force vector C_{ob}; U_b is an additional control to be designed; $D_b(t)$ is a bounded input disturbance. Time invariant diagonal matrices K_{db} and K_{pb} are the $n\times n$ derivative and proportional gain matrix, respectively. By applying control τ_b to the open-loop arm b dynamic equations, the resulting equations of motion become

$$\ddot{E}+K_{db}\dot{E}+K_{pb}E$$
$$=U_b+\bar{H}_b^{*-1}(\Delta H_b^*\ddot{q}_b+\Delta C_{ob}$$
$$-\gamma F_a+D_b(t)) \quad (8)$$

Arranging the previous equation yields

$$\ddot{E}+K_{db}\dot{E}+K_{pb}E$$
$$=U_{b1}+U_{b2}+\bar{H}_b^{*-1}(\Delta H_b^*\ddot{q}_{db}-\Delta H_b^*\ddot{q}_{db}$$
$$+\Delta C_{ob}+\Delta H_b^*\ddot{q}_b-\gamma F_a+D_b(t)) \quad (9)$$

where $U_b=U_{b1}+U_{b2}$. U_{b1} is an additional control for an adaptive control, and U_{b2} is an additional robust control to compensate for the internal coupling forces and input disturbances. In Eq. (6) and Eq. (7), an expression for the joint acceleration is obtained as

$$\ddot{q}_b = H_b^{*-1}(\bar{H}_b^*(N-U_b)+\Delta C_{ob} - \gamma F_a + D_b(t)) \tag{10}$$

where $N=\ddot{q}_{db}-K_{db}\dot{E}-K_{pb}E$. Rewriting Eq. (9) with Eq. (10) yields

$$\begin{aligned}&\ddot{E}+K_{db}\dot{E}+K_{pb}E\\ &=U_{b1}+U_{b2}+\bar{H}_b^{*-1}W_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db})\Delta P_b\\ &\quad-\bar{H}_b^{*-1}\Delta H_b^*\ddot{q}_{db}+\bar{H}_b^{*-1}\Delta H_b^*\ H_b^{*-1}\\ &\quad\{\bar{H}_b^*(N-U_b)+\Delta C_{ob}-\gamma F_a+D_b(t)\}\\ &\quad+\bar{H}_b^{*-1}(D_b(t)-\gamma F_a)\end{aligned} \tag{11}$$

where

$$W_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db})\Delta P_b=\Delta H_b^*\ddot{q}_{db}+\Delta C_{ob},$$
$$\Delta P_b=\bar{P}_b-P_b,$$

and $W_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db})$ is the $n\times r$ matrix of known functions requiring only the feedback of the position and velocity of joint angles, and P_b is a $r\times 1$ vector of an unknown parameters of arm b such as the mass and inertia of the links and the object. $\bar{P}_b$ is a $r\times 1$ vector of a known estimate of parameters of arm b. An additional control for an adaptive controller is designed as

$$U_{b1}=-\bar{H}_b^{*-1}W_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db})\Delta\hat{P}_b$$

where

$$W_b\Delta\hat{P}_b=\Delta\hat{H}_b^*\ddot{q}_{db}+\Delta\hat{C}_{ob} \text{ with}$$
$$\Delta\hat{P}_b=\bar{P}_b-\hat{P}_b$$

also where

$$\Delta\hat{H}_b^*=\bar{H}_b^*-\hat{H}_b^*, \text{ and}$$
$$\Delta\hat{C}_{ob}=\bar{C}_{ob}-\hat{C}_{ob}$$

in which $\hat{P}_b$ is an estimate of unknown parameters of arm b including an object. From now on, $W_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db})$ is expressed as W_b for convenience. Applying U_{b1} to Eq. (11) and arrangingit yield

$$\begin{aligned}&\ddot{E}+K_{db}\dot{E}+K_{pb}E\\ &=\bar{H}_b^{*-1}W_b\tilde{P}_b+U_{b2}\\ &\quad+\Omega_b(q_b,\ \dot{q}_b,\ \ddot{q}_{db},\ F_a,\ t)\end{aligned} \tag{12}$$

where $\tilde{P}_b=\hat{P}_b-P_b$ and

$$\begin{aligned}\Omega_b=\omega_{b1}+\Omega_{b2}(-K_{db}\dot{E})+\Omega_{b2}(-K_{pb}E)\\ +\Omega_{b2}(-U_{b2})\end{aligned} \tag{13}$$

with

$$\begin{aligned}\omega_{b1}&=-(I-\bar{H}_b^{*-1}H_b^*)\ddot{q}_{db}\\ &\quad+(H_b^{*-1}\bar{H}_b^*-I)\ddot{q}_{db}\\ &\quad+(I-\bar{H}_b^{*-1}H_b^*)H_b^{*-1}\Delta\hat{H}_b^*\ddot{q}_{db}\\ &\quad+(H_b^{*-1}-\bar{H}_b^{*-1})(\Delta C_{ob}+\Delta\hat{C}_{ob}\\ &\quad-\gamma F_a+D_b(t))\\ &\quad+\bar{H}_b^{*-1}(D_b(t)-\gamma F_a)\\ \Omega_{b2}&=(H_b^{*-1}\bar{H}_b^*-I)\end{aligned}$$

and

$$\bar{H}_b^{*-1}\Delta H_b^*=I-\bar{H}_b^{*-1}H_b^*$$
$$\bar{H}_b^{*-1}\Delta H_b^*H_b^{*-1}\ \bar{H}_b^*=H_b^{*-1}\bar{H}_b^*-I.$$

Equation (12) represents the closed-loop dynamics with the parametric errors $\bar{H}_b^{*-1}W_b\tilde{P}_b$ and the disturbance term Ω_b including the coupling forces and input disturbances. In order to regulate the internal coupling forces F_a, a force regulator is required such that direct force dynamic equations are derived from arm a dynamics in the joint space. The derivation is shown in the next section.

4. A Force Regulation of Follower Arm via an Inverse Dynamics Control Scheme Under Uncertainty

In Eq. (1), the arm a dynamics are expressed in terms of the joint coordinates. In order to express the arm a dynamics in terms of force dynamics, the arm a dynamics must be expressed in task space beforehand. The joint coordinates can be mapped to the task space by Jacobian matrix as

$$\dot{q}_a=J_a^{-1}\dot{x}_a,$$
$$\ddot{q}_a=\dot{J}_a^{-1}\dot{x}_a+J_a^{-1}\ddot{x}_a \tag{14}$$

With Eq. (13), arm a dynamic equations become,

$$\begin{aligned}\ddot{x}_a=J_aH_a^{-1}(U_a+J_a^TF_a-C_a)\\ -J_a\dot{J}_a^{-1}\dot{x}_a\end{aligned} \tag{15}$$

Eq. (14) represents the dynamics of arm a interacting with arm b where the acceleration and the velocity of the arm a dynamics are expressed in task space. In Eq. (15), the interacting force can be expressed as

$$F_a=K_{pp}(x_a-x_b) \tag{16}$$

where K_{pp} is $n\times n$ diagonal force sensor gain matrix. Also, in Eq. (14), the velocity and acceleration of arm a in task space can be expressed by the first and the second derivative of force vector respectively as.

$$x_a = x_b + K_{pp}^{-1} F_a,$$
$$\dot{x}_a = \dot{x}_b + K_{pp}^{-1} \dot{F}_a,$$
$$\ddot{x}_a = \ddot{x}_b + K_{pp}^{-1} \ddot{F}_a$$

By substituting the above equations into Eq. (15) and arranging them, force dynamic equations for arm a are expressed as

$$\rho_1 K_{pp}^{-1} \ddot{F}_a + \rho_2 K_{pp}^{-1} \dot{F}_a + R = \tau_a + J_a^T F_a - C_a \tag{17}$$

where

$$\rho_1 = H_a J_a^{-1},\ \rho_2 = H_a \dot{J}_a^{-1},$$
$$R = H_a (J_a^{-1} \ddot{x}_b + \dot{J}_a^{-1}\ \dot{x}_b),$$

and the trajectory is well defined such that J_a^{-1} is always existent and bounded. In the force dynamic equations of arm a, the interacting force F_a can be regulated by a direct force control scheme.

In control of arm a, an inverse dynamics control scheme with desired force dynamics is composed as

$$\tau_a = \bar{\rho}_1 K_{pp}^{-1}(M - U_f) + \bar{\rho}_2 K_{pp}^{-1} \dot{F}_a - J_a^T F_a + \bar{C}_a + D_a(t) \tag{18}$$

where

$$M = \ddot{F}_{da} - K_{df} \dot{E}_f - K_{pf} E_f,$$
$$\bar{\rho}_1 = \bar{H}_a J_a^{-1},\ \bar{\rho}_2 = \bar{H}_a \dot{J}_a^{-1}$$

and $E_f = F_a - F_{da}(t)$ in which $F_{da}(t)$ is a bounded desired input signal; $\bar{H}_a$ is a known estimate of the inertia matrix of arm a; $\bar{C}_a$ is a known feedforward estimate of the nonlinear force vector of arm a dynamics C_a; U_f is an additional control to be designed; $D_a(t)$ is a bounded input disturbance, and K_{pf} and K_{df} are the $n \times n$ positive diagonal time-invariant proportional and derivative gain matrix, respectively. Applying the proposed control in Eq. (18) to Eq. (17) yields

$$\rho_1 K_{pp}^{-1} \ddot{F}_a = \bar{\rho}_1 K_{pp}^{-1}(M - U_f) + (\bar{\rho}_2 - \rho) K_{pp}^{-1} \dot{F}_a + \bar{C}_a - C_a - R + D_a(t)$$

Arranging the previous equation yields

$$\ddot{E}_f + K_{df} \dot{E}_f + K_{pf} E_f = U_f + K_{pp} \bar{\rho}_1^{-1} (\Delta\rho_1 K_{pp}^{-1} \ddot{F}_a - R + \Delta\rho_2 K_{pp}^{-1} \dot{F}_a + \Delta C_a + D_a(t))$$

where

$$\Delta\rho_1 = \bar{\rho}_1 - \rho_1,\ \Delta\rho_2 = \bar{\rho}_2 - \rho_2$$

and in $\bar{\rho}_1$, the known estimates are well chosen such that it is invertible and is not near-singular such that $\bar{\rho}_1^{-1}$ is always upper bounded. Based on this condition, arranging the previous equation yields

$$\ddot{E}_f + K_{df} \dot{E}_f + K_{pf} E_f = U_{f1} + U_{f2} + K_{pp} \bar{\rho}_1^{-1} W_f(q_a,\ \dot{q}_a,\ F_a,\ \dot{F}_a,\ \ddot{F}_{da},\ t) \Delta P_f + K_{pp} \bar{\rho}_1^{-1} (\Delta\bar{\rho}_1 K_{pp}^{-1} \ddot{F}_a - \Delta\bar{\rho}_1 K_{pp}^{-1} \ddot{F}_{da} - R + D_a(t)) \tag{19}$$

where

$$W_f \Delta P_f = \Delta\rho_1 K_{pp}^{-1} \ddot{F}_{da} + \Delta\rho_2 K_{pp}^{-1} \dot{F}_a + \Delta C_a,$$
$$\Delta P_f = \bar{P}_f - P_f,\ \Delta C_a = \bar{C}_a - C_a$$

and $U_f = U_{f1} + U_{f2}$, in which U_{f1} is for an adaptive control; and U_{f2} is for a robust control to compensate the internal coupling forces and input disturbances; W_f is the $n \times s$ matrix of the known functions only requiring the feedback of the force and its first derivative; $\bar{P}_f$ is a $s \times 1$ vector of a known estimate of the parameters of arm a, and R is the internal coupling forces between arm a and b including the $\ddot{x}_b$ term. By Eq. (10) and the Jacobian relation, $\ddot{x}_b = \dot{J}_b \dot{q}_b + J_b \ddot{q}_b$, R can be expressed as a function of the first order states. A control for an adaptive controller is designed as

$$U_{f1} = -K_{pp} \bar{\rho}_1^{-1} W_f \Delta\hat{P}_f$$

where

$$W_f \Delta\hat{P}_f = \Delta\hat{\rho}_1 K_{pp}^{-1} \ddot{F}_{da} + \Delta\hat{\rho}_2 K_{pp}^{-1} \dot{F}_a + \Delta\hat{C}_a,$$

also where

$$\Delta\hat{\rho}_1 = \bar{\rho}_1 - \hat{\rho}_1,\ \Delta\hat{\rho}_2 = \bar{\rho}_2 - \hat{\rho}_2,$$
$$\Delta\hat{P}_f = \bar{P}_f - \hat{P}_f,\ \Delta\hat{C}_a = \bar{C}_a - \hat{C}_a$$

and $\hat{P}_f$ is an estimate of the unknown parametersof arm a which is updated by the adaptive control algorithm. Applying the previous control to Eq. (19) yields

$$\ddot{E}_f + K_{df} \dot{E}_f + K_{pf} E_f = K_{pp} \bar{\rho}_1^{-1} W_f \tilde{P}_f + U_{f2} + \Omega_f \tag{20}$$

where

$$\tilde{P}_f = \hat{P}_f - P_f, \text{ and}$$
$$\Omega_f = K_{pp} \bar{\rho}_1^{-1} \{\Delta\rho_1 K_{pp}^{-1} \ddot{F}_a - \Delta\rho_1 K_{pp}^{-1} \ddot{F}_{da} - R + D_a(t)\}. \tag{21}$$

In Eq. (15) and Eq. (18), the second derivative of interacting force is expressed as

$$\ddot{F}_a = K_{pp}\rho_1^{-1}(\Delta\rho_2 K_{pp}^{-1}\dot{F}_a - R + \bar{\rho}_1 K_{pp}^{-1}(M - U_f) - \Delta C_a + D_a(t))$$
$$= K_{pp}\rho_1^{-1}(\Delta\rho_2 K_{pp}^{-1}\dot{F}_a - R + \bar{\rho}_1 K_{pp}^{-1}(M - U_{f1}) - \Delta C_a + D_a(t)) - K_{pp}\rho_1^{-1}\bar{\rho}_1 K_{pp}^{-1} U_{f2}. \quad (22)$$

Substituting Eq. (22) into Eq. (21) and rearranging it yields

$$\Omega_f = \omega_{f1} + \Omega_{f2}(-K_{df}\dot{E}_f) + \Omega_{f2}(-K_{pf}E_f) + \Omega_{f2}(-U_{f2}) \quad (23)$$

where

$$\omega_{f1} = K_{pp}(\rho_1^{-1}\bar{\rho}_1 - I)(\Delta\rho_2 K_{pp}^{-1}\dot{F}_a - R + \bar{\rho}_1 K_{pp}^{-1}\ddot{F}_{da} - \Delta C_a + D_a(t)) + K_{pp}\bar{\rho}_1^{-1}(-\Delta\rho_1 K_{pp}^{-1}\ddot{F}_{da} - R + D_a(t))$$
$$\Omega_{f2} = K_{pp}(H_a^{-1}\bar{H}_a - I)K_{pp}^{-1}$$

with

$$\bar{\rho}_1^{-1}\Delta\bar{\rho}_1 = \rho_1^{-1} - \bar{\rho}_1^{-1}$$
$$\bar{\rho}_1^{-1}\Delta\bar{\rho}_1 K_{pp}^{-1}K_{pp}\rho_1^{-1}\bar{\rho}_1 = \rho_1^{-1}\bar{\rho}_1 - I = H_a^{-1}\bar{H}_a - I$$

5. Adaptive and Robust Controller Design and Its Stability Analysis

Rewriting Eq. (12) and Eq. (20) yields

$$\ddot{E} + K_{db}\dot{E} + K_{pb}E = \bar{H}_b^{*-1}W_b\tilde{P}_b + \Omega_b + U_{b2}$$
$$\ddot{E}_f + K_{df}\dot{E}_f + K_{pf}E_f = K_{pp}\bar{\rho}_1^{-1}W_f\tilde{P}_f + \Omega_f + U_{f2}.$$

The above equations can be expressed as $2n \times 1$ error dynamic equations with nonlinear coupling forces, parameter uncertainties, and bounded input disturbances. Expressing them in a matrix expression as

$$\ddot{X} + C\dot{X} + GX = W\tilde{P} + \Omega + U$$
$$X = \begin{bmatrix} E \\ E_f \end{bmatrix}, \quad C = \begin{bmatrix} K_{db} & 0 \\ 0 & K_{df} \end{bmatrix},$$
$$G = \begin{bmatrix} K_{pb} & 0 \\ 0 & K_{pf} \end{bmatrix}, \quad \tilde{P} = \begin{bmatrix} \tilde{P}_b \\ \tilde{P}_f \end{bmatrix}$$
$$\Omega = \begin{bmatrix} \Omega_b \\ \Omega_f \end{bmatrix}, \quad U = \begin{bmatrix} U_{b2} \\ U_{f2} \end{bmatrix}$$
$$W = \begin{bmatrix} \bar{H}_b^{-1}W_b & 0 \\ 0 & K_{pp}\bar{\rho}_1^{-1}W_f \end{bmatrix} \quad (24)$$

and X is $2n \times 1$ vector. W is $2n \times (r+s)$ matrix of known functions of arm a and b. $\tilde{P}$ is $(r+s) \times 1$ vectors of parameter estimate errors, Ω is $2n \times 1$ vector of bounded nonlinear coupling forces and input disturbances, and U is a robust control to counteract Ω term. Expressing the closed-loop two-arm dynamics in state space form yields

$$\dot{X} = Y,$$
$$\dot{Y} = -CY - GX + W(X, Y)\tilde{P} + \Omega + U \quad (25)$$

Also, a Lyapunov function $V = V(X, Y, \tilde{P})$ is chosen as

$$2V = \langle Y + CX, Y + CX \rangle + \langle Y, Y \rangle + 2\langle GX, X \rangle + \langle \tilde{P}, \Gamma\tilde{P} \rangle \quad (26)$$

where Γ is a $2n \times 2n$ symmetric positive diagonal matrix. The Lyapunov function $V(X, Y, \tilde{P})$ is a convex function and positive scalar because the matrices C and G are defined as positive diagonal matrices. Differentiating the Lyapunov function of the Eq. (26) yields

$$\dot{V} = -\langle CX, GX \rangle - \langle Y, CY \rangle + \langle 2Y + CX, \Omega + U \rangle + \langle 2Y + CX, W\tilde{P} \rangle + \langle \dot{\tilde{P}}, \Gamma\tilde{P} \rangle \quad (27)$$

where the last two terms in Eq. (27) become

$$\langle 2Y + CX, W\tilde{P} \rangle + \langle \dot{\tilde{P}}, \Gamma\tilde{P} \rangle$$
$$= (W\tilde{P})^T(2Y + CX) + \dot{\tilde{P}}^T\Gamma\tilde{P}$$
$$= \tilde{P}^T\{W^T(2Y + CX) + \Gamma\dot{\tilde{P}}\} \quad (28)$$

where in order to eliminate the last two terms which are not negative definite, an adaptive control algorithm is designed as

$$\dot{\tilde{P}} = -\Gamma^{-1}W^T(2Y + CX) \quad (29)$$

where the W^T is a known function of the position and velocity of the joint angles and the force and its first derivative. The adaptive control algorithm is practical since the position and velocity of the joint angles and the force and its first derivative are measurable in reality.

In the third term of the righthand side of Eq. (27),

$$\langle 2Y + CX, \Omega + U \rangle = (2Y + CX)^T(\Omega + U)$$

Ω can be expressed as

$$\Omega = \Omega_1 + \Omega_2(-U) \quad (30)$$

where $\Omega_1 = a_1 + a_2X + a_3Y$ with

$$a_1 = \begin{bmatrix} \omega_{b1} \\ \omega_{f1} \end{bmatrix},$$
$$a_2 = \begin{bmatrix} \Omega_{b2}(-K_{pb}) & 0 \\ 0 & \Omega_{f2}(-K_{pf}) \end{bmatrix},$$

$$a_3=\begin{bmatrix} \Omega_{b2}(-K_{db}) & 0 \\ 0 & \Omega_{f2}(-K_{df}) \end{bmatrix}$$

and Ω_2 is defined in Eq. (13) and Eq. (21) such that

$$\Omega_2=\begin{bmatrix} \Omega_{b2} & 0 \\ 0 & \Omega_{f2} \end{bmatrix}.$$

Also, a robust controller is designed as

$$U=-\frac{\mu}{\|\mu\|}\rho \quad if \ \|\mu\|>\varepsilon$$
$$U=-\frac{\mu}{\varepsilon}\rho \quad if \ \|\mu\|\le\varepsilon \tag{31}$$

where μ is defined as

$$\mu=(2Y+CX),$$

and ε is an arbitrarily small positive constant.

In Eq. (30), assume that there exists positive function ρ as

$$\|\Omega\| \le \|a_1\| + \|a_2\|\,\|X\| + \|a_3\|\,\|Y\| + \|\Omega_2\|\rho=\rho \tag{32}$$

such that

$$\rho=b_1+b_2\|X\|+b_3\|Y\| \tag{33}$$

where

$$b_1=(1-\|\Omega_2\|)^{-1}\|a_1\|,$$
$$b_2=(1-\|\Omega_2\|)^{-1}\|a_2\|,$$
$$b_3=(1-\|\Omega_2\|)^{-1}\|a_3\|$$

where in order to have positive definite function ρ satisfying Eq. (32), two conditions are required as

$$\max\|H_b^{*-1}\bar{H}_b^{*}-I\|<1$$
$$\max\|K_{pp}(H_a^{-1}\bar{H}_a-I)K_{pp}^{-1}\|<1 \tag{34}$$

Rewriting Eq. (27) with the adaptive control in Eq. (29) and the robust control in Eq. (31),

$$\begin{aligned}\dot{V}&=-\langle CX,\,GX\rangle-\langle Y,\,CY\rangle+\langle 2Y+CX,\,\Omega+U\rangle\\&=-X^TCGX-Y^TCY+(2Y+CX)^T(\Omega+U)\le-X^TCGX-Y^TCY\\&\quad+(2Y+CX)^T\left(\frac{\mu}{\|\mu\|}\rho+U\right)\end{aligned} \tag{35}$$

By the control (31), the third term goes to zero for $\|\mu\|>\varepsilon$; if $\|\mu\|<\varepsilon$, the minimum is obtained at $\|\mu\|=(1/2)\varepsilon$ such that

$$\begin{aligned}\dot{V}&\le-\lambda_{cg}X^TX-\lambda_cY^TY+(\varepsilon/4)(b_1+b_2\|X\|+b_3\|Y\|)\\&\le-\lambda_{cg}(\|X\|-\varepsilon b_2/(8\lambda_{cg}))^2\lambda_g\\&\quad(\|Y\|-\varepsilon b_3/(8\lambda_g))^2+d\end{aligned} \tag{36}$$

where $d=(\varepsilon/4)b_1+(\varepsilon b_2)^2/(64\lambda_{cg})+(\varepsilon b_3)^2/(64\lambda_g)$, λ_{cg} and λ_g are the minimum eigen values of CG and C matrices, respectively. In Eq. (36), in order to have the condition for negative semi-definite of the derivative of Lyapunov function, position and velocity ellipsoid are obtained as

$$\|\eta_x\|=\varepsilon b_2/(8\lambda_{cg})+(d/\lambda_{cg})^{1/2}$$
$$\|\eta_y\|=\varepsilon b_3/(8\lambda_g)+(d/\lambda_g)^{1/2} \tag{37}$$

If $\|X\|\ge\eta_x$ and $\|Y\|\ge\eta_y$, then $\dot{V}\le0$ is obtained. The above equations imply that the state vector of the closed-loop two-arm system is confined in a region. The state vector is composed of the error states of the two-arm system such that the boundedness of the error states are guaranteed. Also, η_x and η_y are a function of ε such that the bounded region gets smaller as η_x and η_y gets smaller.

6. Numerical Example

In these numerical examples, a two-arm motion coordination is simulated by the inverse dynamics control scheme in conjunction with the adaptive and robust control. For numerical simulation, dual 3 degree-of-freedom planar arms holding an object with unknown mass and inertia are used with a point mass at the center of each arm as shown in Fig. 1. The test input signals for the leader arm are composed of the composite of three sinusoidal functions with different frequencies. For 10 seconds, two arms are coordinated. The initial and final positions of the mass center are ($X_i=1.38$ m, $Y_i=0.0$ m) and ($X_f=1.7$

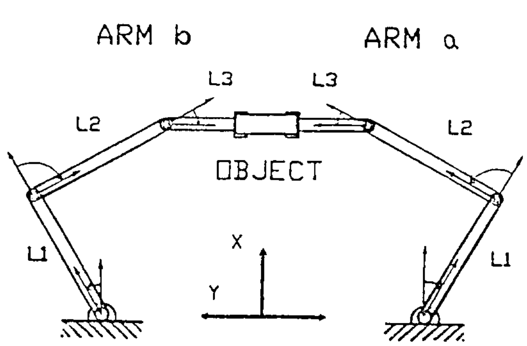

Fig. 1 A dual arm system

m, $Y_f = 1.2$ m). The gains for position and force controllers are assigned $K_{db} = 6$, $K_{pb} = 10$ and for force controller, $K_{fd} = 0.6$, $K_{fp} = 10$, respectively. For arm a and arm b, ε of the robust controller is assigned by the number 0.05 and 0.03 respectively. Adaptive gains for mass and inertia are 14. 5 and 3 in Fig. 2(c) and 33 and 3.7 in Fig. 3(c), respectively. The dimensions of the arms are also shown in Table 1 number 0.05 and 0.03 respectively. The dimensions of the arms are also shown in Table 1.

In the Figs. 2(a), 2(b), 3(a), and 3(b), the upper part represents the force regulation between the two robot arms, and the lower part represents the input command following of the leader arm. In Figs. 2(c) and 3(c), the dashed lines represent the actual mass and inertia, and the solid and dotted lines represent their estimates. Tz, Fx, Fy represent torque about z-axis, force along x-axis, and force along y-axis applied on the end effector of arm a, respectively. Ex and Ey are the position errors along x-axis and y-axis between desired and actual trajectory.

The simulation plots are composed of two sets of cases : first, Figs. 2(a), 2(b), and 2(c), show the cases in which a two-arm system coordinates an object with pulse shaped input disturbances between 2 and 3 seconds after the two-arm is coordinated under an initially unknown mass and inertia of the object. ; second, Figs. 3(a), 3(b), and 3(c) show the cases in which a two-arm system coordinates the object with an initially known mass and inertia but has an unknown change of the mass and inertia after 2 seconds the two-arm is coordinated. In Figs. 2(a) and 3(a), only a force/position control based on an inverse dynamics control is applied to coordinate the two-arm system. Due to these parametric uncertainties and input disturbances, the plots show considerable coordination errors of the leader arm and interacting forces between two arms. When an adaptive and robust control in conjunction with a computed torque control is applied,

Table 1 Demension of a dual arm system

Arm, Obj.	Len. of arm	Mass	Moment of Ine.
L_1	1 m	2 kg	0.2 kg-m^2
L_2	1 m	2 kg	0.2 kg-m^2
L_3	0.5 m	1 kg	0.1 kg-m^2
L_0	0.134 m	20 kg	3 kg-m^2

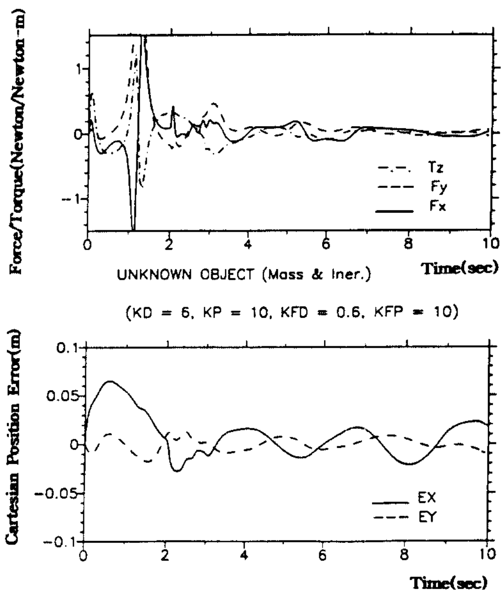

Fig. 2 (a) Force/position control under disturbances

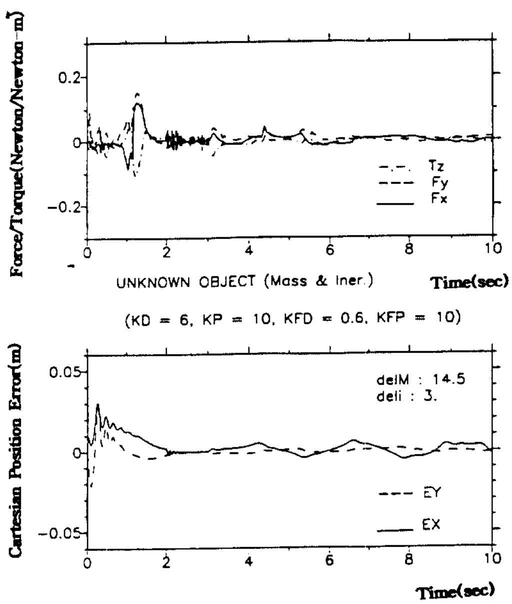

Fig. 2 (b) Force/position control with an adaptive and robust control under disturbances

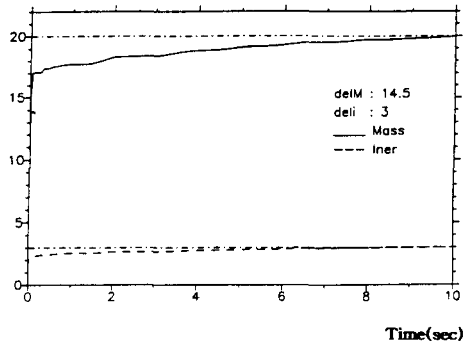

Fig. 2 (c) Parameter estimation by an adaptive control under disturbances

the system shows a good response which reduces position errors of the leader arm and interacting-forces significantly. The response of the closed-loop system is shown in Figs. 2(b) and 3(b). Also, the proposed controller shows a robust response to the input disturbances as shown in Fig. 2(b). In Figs. 2(c) and 3(c), parameter estimations of the actual values of the unknown and changed mass and inertia are shown respectively by the proposed adaptive control scheme.

According to the above numerical simulation results, the adaptive and robust control scheme in

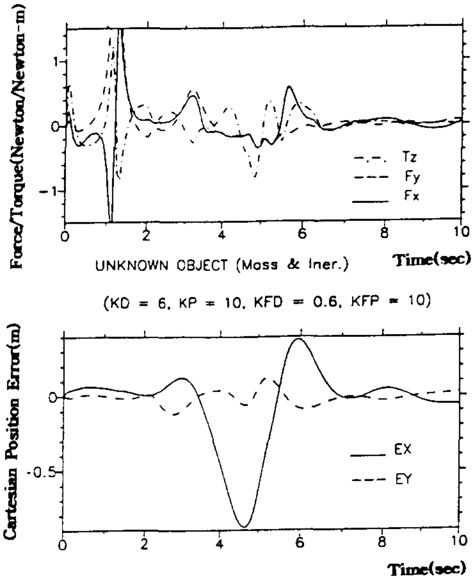

Fig. 3 (a) Force/position control under a step change of mass and inertia

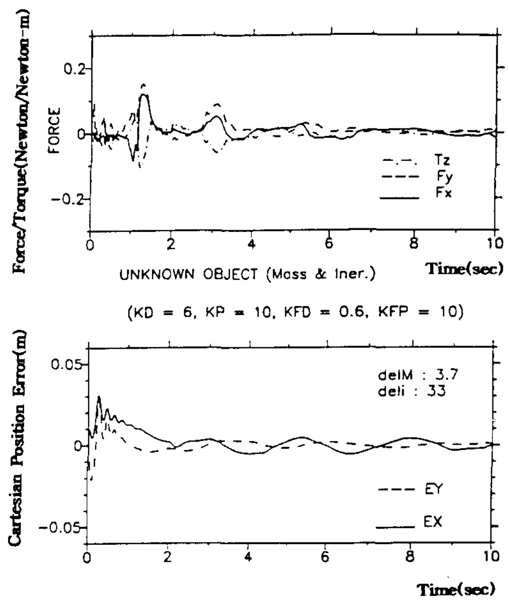

Fig. 3 (b) Force/position control with an adaptive and robust control under a step change of mass and inertia

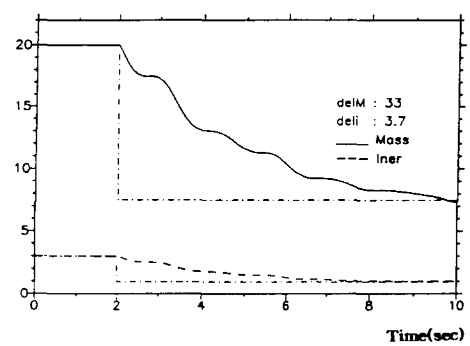

Fig. 3 (c) Parameter estimation of the step changed mass and inertia by an adaptive control under disturbances

conjunction with the inverse dynamics control shows a good motion coordination of the two-arm system under parametric uncertainties. Also, the control scheme shows a robust response to the coupling forces and input disturbances while estimating the actual values of the unknown parameters.

7. Conclusion

A force/position control scheme based on an inverse dynamics control scheme in conjunction with a robust and adaptive control scheme is presented for motion coordination of a two-cooperating robot arm. The adaptive control algorithm is practical since it does not require the feedback of the acceleration of the joint angle and the second derivative of the forces. Also, the robust control scheme is devised to counteract disturbances such as coupling forces between two arms, and bounded input disturbances. The robust and the adaptive control schemes are derived based on a devised Lyapunov function. The boundedness of the state errors are guaranteed by the devised control scheme. Numerical simulation is shown to validate the results of the proposed control schemes. According to the simulation result, unknown parameters of two-arm system is estimated well by the adaptive control scheme. Also, by the robust control, the disturbances are counteracted significantly. In this way, the proposed control scheme shows a good trajectory following.

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